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Scalar a_0-meson contributions to radiative $\omega \to \pi^0\eta\gamma$ and $\rho^0 \to \pi^0\eta\gamma$ decays

A. Gokalp[a], Y. Sarac, O. Yilmaz[b]

Physics Department, Middle East Technical University, 06531 Ankara, Turkey

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Abstract. We study the radiative decays $\omega \to \eta\pi^0\gamma$ and $\rho^0 \to \eta\pi^0\gamma$ within the framework of a phenomenological approach in which the contributions of the intermediate a_0-meson and intermediate vector meson states are considered. We conclude that although the a_0-meson intermediate state amplitude makes a small contribution to $\omega \to \eta\pi^0\gamma$ decay, it makes a substantial contribution to $\rho^0 \to \eta\pi^0\gamma$ decay.

1 Introduction

The light scalar mesons with the vacuum quantum numbers $J^{PC} = 0^{++}$ have fundamental importance in understanding low energy QCD phenomenology. They also have very interesting and peculiar properties from the hadron spectroscopy point of view. In particular, the nature and the quark substructure of the two best known scalar mesons, the isoscalar $f_0(980)$ and isovector $a_0(980)$, have not been established yet; the question whether they are conventional $q\overline{q}$ states [1], $K\overline{K}$ molecules [2], or multiquark exotic $q^2\overline{q}^2$ states [3] has been a subject of controversy. Besides the questions of their nature and structure, since they are relevant hadronic degrees of freedom, the role of scalar mesons in hadronic processes should be investigated.

The radiative decay processes of the type $V^0 \to P^0P^0\gamma$ where V and P belong to the lowest multiplets of vector (V) and pseudoscalar (P) mesons have been a subject of continuous interest both theoretically and experimentally. Since only neutral particles are involved, the bremsstrahlung process does not contribute, and this fact makes it possible to study the interesting mechanisms of structural radiation. However, as a result, such decays have small branching ratios. Nevertheless, with the advent of high-luminosity, low energy e^+e^- machines such as at Novosibirsk and Frascati it will be possible to study these rare decays experimentally. These studies will offer the possibility of testing the theoretical ideas about the interesting mechanisms of structural radiation of these decays, as well as shedding light on the structure of the intermediate states involved in these decays.

The radiative decays of the ρ^0- and ω-mesons into a single photon and pseudoscalar π^0- and η-mesons as well as other radiative vector meson decays were studied by Fajfer and Oakes [4], using a low energy effective Lagrangian approach with gauged Wess–Zumino terms. They neglected scalar meson contributions, and obtained the branching ratios for these decays as $\mathrm{BR}(\omega \to \pi^0\eta\gamma) = 6.26 \times 10^{-6}$ and $\mathrm{BR}(\rho^0 \to \pi^0\eta\gamma) = 3.98 \times 10^{-6}$. The contributions of intermediate vector mesons to the decays $V^0 \to P^0P^0\gamma$ were later considered by Bramon et al. [5] using standard Lagrangians obeying the $SU(3)$ symmetry. Their results for the decay rates and the branching ratios of the decays $\omega \to \pi^0\eta\gamma$ and $\rho^0 \to \pi^0\eta\gamma$ were $\Gamma(\omega \to \pi^0\eta\gamma) = 1.39\,\mathrm{eV}$, $\mathrm{BR}(\omega \to \pi^0\eta\gamma) = 1.6\times10^{-7}$ and $\Gamma(\rho^0 \to \pi^0\eta\gamma) = 0.061\,\mathrm{eV}$, $\mathrm{BR}(\rho^0 \to \pi^0\eta\gamma) = 4 \times 10^{-10}$. These results were not in agreement with the numerical predictions of Fajfer and Oakes [4], even if the initial expressions for the Lagrangians were the same. Bramon et al. later considered these decays among other similar vector meson decays within the framework of chiral effective Lagrangians, and using chiral perturbation theory they calculated the decay rates of the type $V^0 \to P^0P^0\gamma$ at the one-loop level, including both $\pi\pi$ and $K\overline{K}$ intermediate loops. They showed that the one-loop contributions are finite and to this order no counterterms are required. They obtained for the $K\overline{K}$ loop contributions to the decay rates of the decays $\omega \to \pi^0\eta\gamma$ and $\rho^0 \to \pi^0\eta\gamma$ the results $\Gamma(\omega \to \pi^0\eta\gamma)_K = 0.013\,\mathrm{eV}$ and $\Gamma(\rho^0 \to \pi^0\eta\gamma)_K = 0.006\,\mathrm{eV}$ with the $\pi\pi$ loop contributions vanishing in the good isospin limit. Therefore, these decays proceed through intermediate vector meson states and the $K\overline{K}$ loop contributions are one or two orders of magnitude smaller and the dominant $\pi\pi$ loops are forbidden in these decays due to isospin symmetry.

In this work, we study the contribution of the scalar $a_0(980)$ intermediate meson state in addition to the contributions of intermediate vector meson states to the $\omega \to \pi^0\eta\gamma$ and $\rho^0 \to \pi^0\eta\gamma$ radiative decays. We follow a phenomenological approach, and we attempt to calculate the

[a] e-mail: agokalp@metu.edu.tr
[b] e-mail: oyilmaz@metu.edu.tr

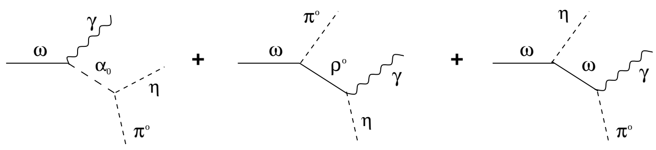

Fig. 1. Feynman diagrams for the decay $\omega \to \pi^0\eta\gamma$

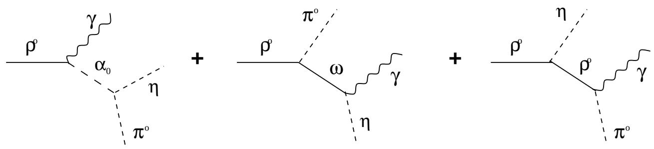

Fig. 2. Feynman diagrams for the decay $\rho^0 \to \pi^0\eta\gamma$

decay rates of these decays by considering a_0-pole and vector meson-pole diagrams using effective Lagrangians where we relate the coupling constants we use to the experimentally measured quantities.

2 Formalism

Our calculation is based on the Feynman diagrams shown in Fig. 1 for $\omega \to \pi^0\eta\gamma$ decay and on those shown in Fig. 2 for $\rho^0 \to \pi^0\eta\gamma$ decay. We describe the $\omega\rho\pi$-vertex by the effective Lagrangian [7]

$$\mathcal{L}^{\text{eff.}}_{\omega\rho\pi} = g_{\omega\rho\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\partial_\alpha\boldsymbol{\rho_\beta}\cdot\boldsymbol{\pi}, \quad (1)$$

which also conventionally defines the coupling constant $g_{\omega\rho\pi}$. Since there is no phase space to measure an $\omega \to \rho\pi$ transition, this coupling constant should be extracted from theoretical models. Vector meson dominance and current-field identities [8] give $g_{\omega\rho\pi} \approx 12\,\text{GeV}^{-1}$, while an approximate $SU(3)$ symmetry suggests $g_{\omega\rho\pi} \approx 16\,\text{GeV}^{-1}$ [9]. On the other hand, QCD sum rule calculations obtain the value $g_{\omega\rho\pi} \approx (15-17)\,\text{GeV}^{-1}$ [7], and the light cone QCD sum rules method extracts the value $g_{\omega\rho\pi} \approx 15\,\text{GeV}^{-1}$ [10]. Recently, the QCD sum rules method for the polarization operator in an external field yielded the value $g_{\omega\rho\pi} \approx 16\,\text{GeV}^{-1}$ [11]. In this work, we use the coupling constant $g_{\omega\rho\pi} = 15\,\text{GeV}^{-1}$. For the $V\varphi\gamma$-vertices, where V denotes ω or ρ^0 and φ denotes π^0 or η, we use the effective Lagrangian [12]

$$\mathcal{L}^{\text{eff.}}_{V\varphi\gamma} = g_{V\phi\gamma}\epsilon^{\mu\nu\alpha\beta}\partial_\mu V_\nu\partial_\alpha A_\beta\varphi. \quad (2)$$

Here, φ is the pseudoscalar field π^0 or η, V_μ is the appropriate vector meson field ω_μ or ρ_μ, and A_μ is the photon field. The coupling constants $g_{V\pi\gamma}$ and $g_{V\eta\gamma}$ are obtained from the experimental partial widths [13] of the vector meson radiative decays $V \to \pi^0\gamma$ and $V \to \eta\gamma$. We obtain in this way for these coupling constants the values $g_{\rho\pi\gamma} = 0.274\,\text{GeV}^{-1}$, $g_{\rho\eta\gamma} = 0.461\,\text{GeV}^{-1}$, $g_{\omega\pi\gamma} = 0.706\,\text{GeV}^{-1}$ and $g_{\omega\eta\gamma} = 0.155\,\text{GeV}^{-1}$. We describe the $\rho\rho\eta$- and $\omega\omega\eta$-vertices by the effective Lagrangian

$$\mathcal{L}^{\text{eff.}}_{VV\eta} = g_{VV\eta}\epsilon^{\mu\nu\alpha\beta}\partial_\mu V_\nu V_\alpha\partial_\beta\eta. \quad (3)$$

This Lagrangian was derived from a chiral $SU(3)$ Lagrangian by Klingl et al. [14], and they obtained the coupling constants $g_{VV\eta}$ by utilizing the experimental decay widths of the decays $\omega \to 3\pi$ and $\phi \to 3\pi$ [13]; they found $g_{\omega\omega\eta} = g_{\rho\rho\eta} = 2.624\,\text{GeV}^{-1}$. This result agrees with the previous analyses [15,16]. For the $a_0\pi\eta$-vertex we use the effective Lagrangian

$$\mathcal{L}^{\text{eff.}}_{a_0\pi\eta} = g_1\boldsymbol{a_0}\cdot\partial_\mu\boldsymbol{\pi}\partial^\mu\eta - g_2\boldsymbol{a_0}\cdot\boldsymbol{\pi}\eta. \quad (4)$$

This Lagrangian was derived by Ecker et al. [17] in their study of the role of resonances in chiral perturbation theory. They determined the scalar coupling constants by considering the resonance contributions to the coupling constants of the $O(p^4)$ effective chiral Lagrangian involving pseudoscalar fields. Using their result we can express the coupling constants g_1 and g_2 by $g_1 = 6.042 \times 10^{-3}\,\text{MeV}^{-1}$ and $g_2 = 1.445 \times 10^2\,\text{MeV}$. If we use the effective Lagrangian $\mathcal{L}^{\text{eff.}}_{a_0\pi\eta}$ and these values of the coupling constants, then the decay rate of the decay $a_0 \to \pi\eta$ can be calculated to be $\Gamma(a_0 \to \pi\eta) = 59\,\text{MeV}$ [17]. This result compares well with the experimental results $\Gamma(a_0 \to \pi\eta) = 54 \pm 4\,\text{MeV}$ [18] and $\Gamma(a_0 \to \pi\eta) = 69 \pm 11\,\text{MeV}$ [19]. Therefore, in our work we use these values of the coupling constants g_1 and g_2. We describe the $a_0V\gamma$-vertex by the effective Lagrangian

$$\mathcal{L}^{\text{eff.}}_{a_0V\gamma} = g_{a_0V\gamma}\left[\partial^\alpha V^\beta\partial_\alpha A_\beta - \partial^\alpha V^\beta\partial_\beta A_\alpha\right]a_0. \quad (5)$$

Since there are no direct experimental results relating to the $a_0V\gamma$-vertex that will allow us to determine the coupling constants $\text{g}_{a_0V\gamma}$, we use the QCD sum rule method to estimate the coupling constants $g_{Va_0\gamma}$. The details of our calculation are presented elsewhere [20]; here let us quote the results we obtained for the coupling constants $g_{Va_0\gamma}$: $g_{\rho a_0\gamma} = 1.69 \pm 0.39\,\text{GeV}^{-1}$ and $g_{\omega a_0\gamma} = 0.58 \pm 0.13\,\text{GeV}^{-1}$. In our calculation of the invariant amplitudes for the decays $\omega \to \pi^0\eta\gamma$ and $\rho^0 \to \pi^0\eta\gamma$, in the ρ^0-meson and a_0-meson propagators we make the replacement $M \to M - (1/2)\text{i}\Gamma$. For the ρ^0-meson we use the experimental value of the width $\Gamma_\rho = 150.2\,\text{MeV}$ [13], and for the a_0-meson following our discussion of the $a_0\pi\eta$-vertex we use $\Gamma_{a_0} = 59\,\text{MeV}$. Furthermore, we use the energy dependent widths for the ρ^0-meson [21] and for the a_0-meson [22] given by

$$\Gamma_\rho(q^2) = \Gamma_\rho\left(\frac{q^2 - 4M_\pi^2}{M_\rho^2 - 4M_\pi^2}\right)^{3/2}\frac{M_\rho}{\sqrt{q^2}} \quad (6)$$

and

$$\Gamma_{a_0}(q^2) = \Gamma_{a_0}\left(\frac{q^2 - 4M_\pi^2}{M_{a_0}^2 - 4M_\pi^2}\right)^{1/2}\frac{M_{a_0}^2}{q^2}, \quad (7)$$

where q^2 is the four momentum squared of the virtual ρ^0- or a_0-meson in the intermediate state.

In terms of the invariant amplitude $\mathcal{M}(\text{E}_\gamma, \text{E}_1)$, the differential decay probability of $V^0 \to \pi^0\eta\gamma$ decay for an unpolarized V^0-meson, with V^0 denoting ω or ρ^0, is then given at rest by

$$\frac{\text{d}\Gamma}{\text{d}E_\gamma\text{d}E_1} = \frac{1}{(2\pi)^3}\frac{1}{8M_V}\mid\mathcal{M}\mid^2, \quad (8)$$

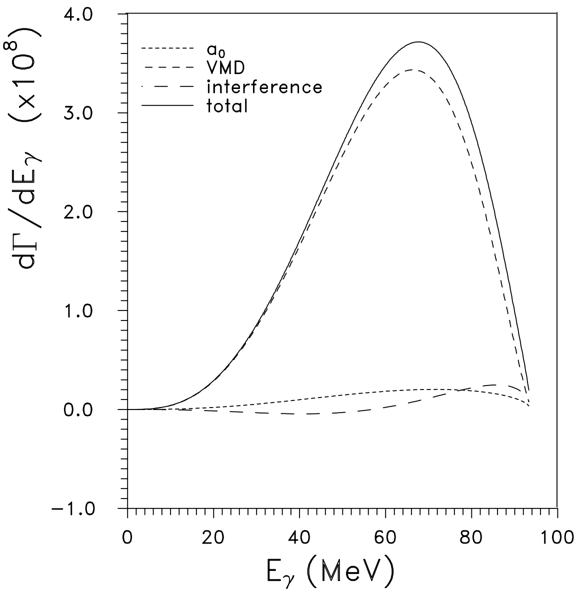

Fig. 3. The photon spectra for the decay rate of the decay $\omega \to \pi^0\eta\gamma$

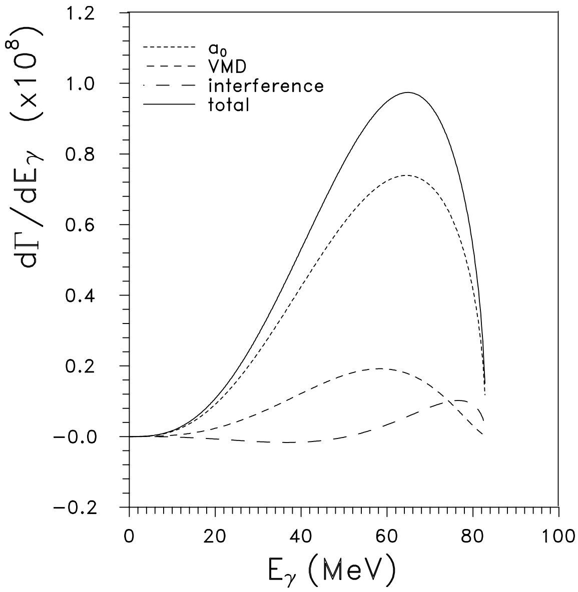

Fig. 4. The photon spectra for the decay rate of the decay $\rho^0 \to \pi^0\eta\gamma$

where E_γ and E_1 are the photon and pion energies, respectively. We perform an average over the spin states of the V^0-meson and sum over the polarization states of the photon. The decay width $\Gamma(V^0 \to \pi^0\eta\gamma)$ is then obtained by integration:

$$\Gamma = \int_{E_{\gamma,\mathrm{min.}}}^{E_{\gamma,\mathrm{max.}}} \mathrm{d}E_\gamma \int_{E_{1,\mathrm{min.}}}^{E_{1,\mathrm{max.}}} \mathrm{d}E_1 \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma \mathrm{d}E_1}. \quad (9)$$

The minimum photon energy is $E_{\gamma,\mathrm{min.}} = 0$ and the maximum photon energy is given by $E_{\gamma,\mathrm{max.}} = (M_V^2 - (M_\pi + M_\eta)^2)/2M_V$. The maximum and minimum values for the pion energy E_1 are given by

$$\frac{1}{2(2E_\gamma M_V - M_V^2)} \Bigg[-2E_\gamma^2 M_V - M_V(M_V^2 + M_\pi^2 - M_\eta^2) + E_\gamma(3M_V^2 + M_\pi^2 - M_\eta^2) \pm E_\gamma \Big(4E_\gamma^2 M_V^2 + M_V^4 + (M_\pi^2 - M_\eta^2)^2 - 2M_V^2(M_\pi^2 + M_\eta^2) + 4E_\gamma M_V(-M_V^2 + M_\pi^2 + M_\eta^2) \Big)^{1/2} \Bigg].$$

3 Results and discussion

The photon spectra for the decay rate of the decay $\omega \to \pi^0\eta\gamma$ is plotted in Fig. 3 and that for the decay $\rho^0 \to \pi^0\eta\gamma$ is plotted in Fig. 4. In these figures, the contributions of the a_0-meson intermediate state amplitude, the vector meson dominance amplitude as well as the contributions of interference terms are indicated. Comparison of Figs. 3 and 4 reveals that the a_0-meson amplitude contribution to the overall decay rate for $\rho^0 \to \pi^0\eta\gamma$ decay is quite significant, and it is larger than the contribution of the vector meson intermediate state amplitude. However, for the $\omega \to \pi^0\eta\gamma$ decay the contribution of the a_0-meson intermediate state amplitude is quite small and for this decay the overall decay rate is determined almost exclusively by the vector meson intermediate state amplitude. The contributions of different amplitudes to the decay rate of the decay $\omega \to \pi^0\eta\gamma$ are $\Gamma(\omega \to \pi^0\eta\gamma)_{\mathrm{VMD}} = 1.51\,\mathrm{eV}$, $\Gamma(\omega \to \pi^0\eta\gamma)_{a_0} = 0.07\,\mathrm{eV}$, $\Gamma(\omega \to \pi^0\eta\gamma)_{\mathrm{int.}} = 0.02\,\mathrm{eV}$, resulting in the total rate $\Gamma(\omega \to \pi^0\eta\gamma) = 1.62\,\mathrm{eV}$ and the total branching ratio $\mathrm{BR}(\omega \to \pi^0\eta\gamma)) = 1.92 \times 10^{-7}$. If we use a constant width for the ρ^0-meson we obtain the branching ratio $\mathrm{BR}(\omega \to \pi^0\eta\gamma)) = 1.64 \times 10^{-7}$, and the contribution to the decay rate coming from the vector meson dominance amplitude $\Gamma(\omega \to \pi^0\eta\gamma)_{\mathrm{VMD}} = 1.30\,\mathrm{eV}$. Our result for the vector meson intermediate state is in agreement with that of Bramon et al. [5]. For the $\rho^0 \to \pi^0\eta\gamma$ decay the results we obtain are $\Gamma(\rho^0 \to \pi^0\eta\gamma)_{\mathrm{VMD}} = 0.08\,\mathrm{eV}$, $\Gamma(\rho^0 \to \pi^0\eta\gamma)_{a_0} = 0.32\,\mathrm{eV}$, $\Gamma(\rho^0 \to \pi^0\eta\gamma)_{\mathrm{int.}} = 0.03\,\mathrm{eV}$ and the total rate $\Gamma(\rho^0 \to \pi^0\eta\gamma) = 0.43\,\mathrm{eV}$ with the total branching ratio $\mathrm{BR}(\rho^0 \to \pi^0\eta\gamma)) = 2.9 \times 10^{-9}$. If a constant width for the intermediate ρ^0-meson is used, then the resulting branching ratio is $\mathrm{BR}(\rho^0 \to \pi^0\eta\gamma)) = 2.7 \times 10^{-9}$, and the decay rate resulting from the intermediate vector meson state is $\Gamma(\rho^0 \to \pi^0\eta\gamma)_{\mathrm{VMD}} = 0.07\,\mathrm{eV}$. In this case also our result for the vector meson intermediate state contribution agrees with the result of Bramon et al. [5], but we disagree with the result of Fajfer and Oakes [4]. Although the a_0-meson intermediate state makes a substantial contribution to the decay rate of $\rho^0 \to \pi^0\eta\gamma$ and increases it by more than a factor of five as compared to the contribution of the vector meson intermediate state, the overall branching ratio of this decay is still very small.

Achasov and Gubin [23] argued that the reaction $e^+e^- \to \gamma\pi^0\pi^0(\eta)$ would give a good opportunity for observing the scalar $f_0(980)$- and $a_0(980)$-mesons. In their analysis they showed that the processes $e^+e^- \to V^0 \to \pi^0 V'^0 \to \gamma\pi^0\eta$ and $e^+e^- \to V^0 \to \eta V'^0 \to \gamma\pi^0\eta$ are small backgrounds for the reaction $e^+e^- \to \gamma a_0 \to \gamma\pi^0\eta$. Our results for the decay $\rho^0 \to \eta\pi^0\gamma$ are in agreement with their conclusions. On the other hand, Bramon et al. [5] in their analysis of intermediate vector meson contributions to $V^0 \to P^0P^0\gamma$ decays mentioned that in the ρ–ω region, which is below the mass of the established scalar mesons $f_0(980)$ and $a_0(980)$, the contributions coming from the scalar mesons to the decay rates $V^0 \to \pi^0\pi^0(\eta)\gamma$ are expected to be rather suppressed as compared to the contributions coming from the vector meson dominance amplitudes, and therefore these decays proceed mainly through the mechanism of vector meson dominance. However, our analysis of the decay $\rho^0 \to \pi^0\eta\gamma$ shows that this may not be the case, and the a_0-meson intermediate state amplitude makes substantial contributions to the decay rate of this decay.

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